Properties of Logarithms – Expanding Logarithms

What are the Properties of Logarithms?

The properties of logarithms are very similar to the properties of exponents because as we have seen before every exponential equation can be written in logarithmic form and vice versa.

Properties for Expanding Logarithms

There are 5 properties that are frequently used for expanding logarithms. These properties are summarized in the table below. When applying the properties of logarithms in the examples shown below and in future examples, the properties will be referred to by number.

Properties for Expanding Logarithms

Property 1: $\log_a 1 = 0$ – Zero-Exponent Rule

Property 2: $\log_a a = 1$

Property 3: $\log_a (xy) = \log_a x + \log_a y$ – Product Rule

Property 4: $\log_a \left(\dfrac{x}{y} \right) = \log_a x - \log_a y$ – Quotient Rule

Property 5: $\log_a x^y = y \log_a x$ – Power Rule

To help see where one of the properties comes from let's look at one of the properties of exponents. If we start with the zero-exponent rule that states $a^0 = 1$ (or that any number raised to the zero power will equal one) and we write this property in logarithmic form, we get $0 = \log_a 1$ or $\log_a 1 = 0$. This is property number 1 which says that log of 1 will always equal zero no matter what the base is. If we went through and rewrote each of the properties of exponents we would get the properties of logarithms shown above.

When we are expanding logarithms there is not a specific order in which these properties must be applied, but some guidelines are listed below.

Guideline for Expanding Logarithms
- Rewrite any radicals using rational exponents (fractions).
- Apply Property 3 or 4 to rewrite the logarithm as addition and subtract instead of multiplication and division.
- Apply Property 5 to move the exponents out front of the logarithms.
- Apply Property 1 or 2 to simplify the logarithms.

Examples – Now let's use the properties of logarithms to expand logarithms.

Example 1: Use the properties of logarithms to expand $\log_3\left(x^2y^7\right)$.

$$\log_3\left(x^2y^7\right) = \log_3 x^2 + \log_3 y^7$$ Use property 3 to rewrite the multiplication as addition.

$$= 2\log_3 x + 7\log_3 y$$ Use Property 5 to move the exponents out front.

Thus, $\log_3\left(x^2y^7\right) = 2\log_3 x + 7\log_3 y$.

Example 2: Use the properties of logarithms to expand $\log_8\left(\dfrac{\sqrt{x}}{y^3}\right)$.

$$\log_8\left(\frac{\sqrt{x}}{y^3}\right) = \log_8\left(\frac{x^{\frac{1}{2}}}{y^3}\right)$$ Rewrite the radical using rational exponents (fractions).

$$= \log_8 x^{\frac{1}{2}} - \log_8 y^3$$ Use property 4 to rewrite the division as subtraction.

$$= \frac{1}{2}\log_8 x - 3\log_8 y$$ Use Property 5 to move the exponents out front.

Thus, $\log_8\left(\dfrac{\sqrt{x}}{y^3}\right) = \dfrac{1}{2}\log_8 - 3\log_8 y$.

Example 3: Use the properties of logarithms to expand $\log_4\left(\dfrac{4x}{y^9}\right)$.

$$\log_4\left(\frac{4x}{y^9}\right) = \log_4 4 + \log_4 x - \log_4 y^9$$ Use properties 3 and 4 to rewrite the multiplication as addition and the division as subtraction. Note that 4x means 4 times x which is why Property 3 has been used to rewrite the logarithm using addition.

$$= \log_4 4 + \log_4 x - 9\log_4 y$$ Use Property 5 to move the exponents out front.

$$= 1 + \log_4 x - 9\log_4 y$$ Use Property 2 to simplify the logarithm.

Thus, $\log_4\left(\dfrac{4x}{y^9}\right) = 1 + \log_4 x - 9\log_4 y$.

Example 4: Use the properties of logarithms to expand $\log\left(\dfrac{\sqrt[4]{x^3}\,y^7}{z^8}\right)$.

$$\log\left(\frac{\sqrt[4]{x^3}\,y^7}{z^8}\right) = \log\left(\frac{x^{3/4}\,y^7}{z^8}\right)$$

Rewrite the radical using rational exponents (fractions).

$$= \log x^{3/4} + \log y^7 - \log z^8$$

Use properties 3 and 4 to rewrite the multiplication as addition and the division as subtraction.

$$= \frac{3}{4}\log x + 7\log y + 8\log z$$

Use Property 5 to move the exponents out front.

Thus, $\log\left(\dfrac{\sqrt[4]{x^3}\,y^7}{z^8}\right) = \dfrac{3}{4}\log + 7\log y - 8\log z$.

Example 5: Use the properties of logarithms to expand $\ln\left(\dfrac{x^5}{y^2 z^7}\right)$.

$$\ln\left(\frac{x^5}{y^2 z^7}\right) = \ln x^5 - \ln y^2 - \ln z^7$$

Use property 4 to rewrite the division as subtraction. Since both the y and z terms are in the denominator of the fraction, they are each considered to be division and this makes each of them into subtraction when expanded.

$$= 5\ln x - 2\ln y - 7\ln z$$

Use Property 5 to move the exponents out front.

Thus, $\ln\left(\dfrac{x^5}{y^2 z^7}\right) = 5\ln x - 2\ln y - 7\ln z$.

Note: The answer to example 5 also could have been written as $5\ln x - (2\ln y + 7\ln z)$. These answers are the same because if we distribute the negative sign in front of the parenthesis we would have gotten the answer shown above. How the answer is written is a matter of personal preference, I prefer to write the answer as $5\ln x - 2\ln y - 7\ln z$ because if the variables, in this case y and z, are in the denominator I think of it as division which leads to subtraction when I expand the logarithm. For me it is easier to think of this as two division problems rather than the division of one multiplication problem.

Addition Examples

If you would like to see more examples of expanding logarithm, just click on the link below.

Practice Problems

Now it is your turn to try a few practice problems on your own. Work on each of the problems below and then click on the link at the end to check your answers.

Problem 1: Use the properties of logarithms to expand $\log_6\left(\dfrac{\sqrt[5]{x^3}}{y^9}\right)$.

Problem 2: Use the properties of logarithms to expand $\log\left(\dfrac{10\sqrt{x}}{y^5}\right)$.

Problem 3: Use the properties of logarithms to expand $\log_8\left(\dfrac{x^{14}y^7}{z^{15}}\right)$.

Problem 4: Use the properties of logarithms to expand $\ln\left(7x^3y^8\right)$.

Problem 5: Use the properties of logarithms to expand $\log_2\left(\dfrac{1}{x^5y^3}\right)$.

Problem 6: Use the properties of logarithms to expand $\log_9\left(\dfrac{\sqrt[6]{x^5}}{y^3(z-2)^7}\right)$.