Introduction to Polynomial Functions

Standard Form: degree

$$f(x) = \underbrace{a_n}_{\text{lead coefficent}} x^{\boxed{n}} + a_{n-1}x^{\underline{n-1}} + \ldots a_1 x + \underline{\underline{a_0}}$$

constant

*Exponents are whole numbers   no fractions
no negative

*Coefficients are real numbers   no "i"

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

$f(x) = 7 - 6x^2 - 5x \rightarrow \underline{\text{yes}}$

$f(x) = -6x^2 - 5x + 7$

deg = 2 quadratic function

lc = −6

$f(x) = x + 2x^{-2} + 9 \rightarrow \underline{\underline{\text{no}}}$

$f(x) = x^3 - 6x + 3x^4 \rightarrow \underline{\text{yes}}$

$f(x) = 3x^4 + x^3 - 6x$

deg = 4 polynomial

lc = 3

Graphs of Polynomials – using the degree

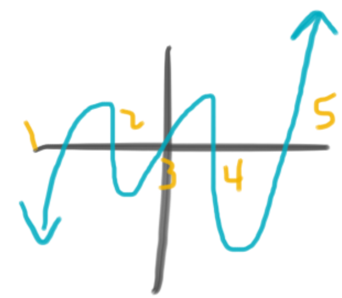
$d=1$

$d=2$

$d=3$

$d=4$

$d=5$



$d=6$

\# of direction changes = degree

End Behavior of Polynomials

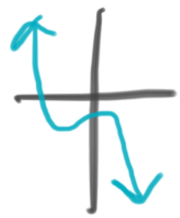
how the leading coefficient affects the graph

Degree: ODD goes in diff direction
Leading Coefficient: POSITIVE end up

$x \to \infty, f(x) \to \infty$

$x \to -\infty, f(x) \to -\infty$

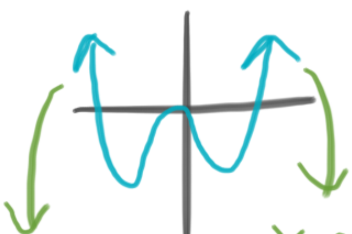
Degree: ODD goes in diff direction
Leading Coefficient: NEGATIVE end down



$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to -\infty$

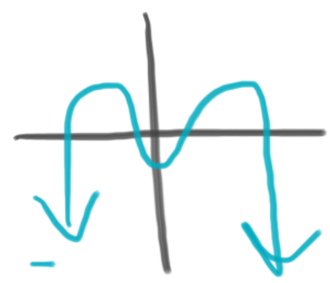
Degree: EVEN goes in the same direction
Leading Coefficient: POSITIVE end up



$x \to -\infty$
$f(x) \to \infty$

$x \to \infty$
$f(x) \to \infty$

Degree: EVEN – goes in the same direction
Leading Coefficient: NEGATIVE, end down



$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to -\infty$

Describe the end behavior:

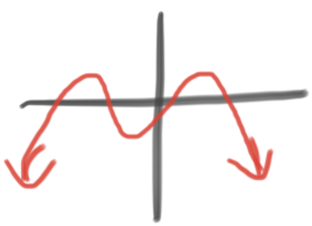
$f(x) = -2x^4 + 2x^2 - 5$

degree = 4 (even)

* 4 direction changes
* end in same direction

leading coeff = -2

* end down



$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to -\infty$

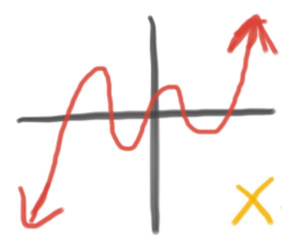
$f(x) = x^5 - 7$

deg = 5 (odd)

* 5 directions
* end in opp directions

lead coeff = 1

* right side will be UP



$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to \infty$

Evaluate f(x) when x = 3.

$$f(x) = 2x^4 - 8x^2 + 5x - 7$$

$$f(3) = 2(3)^4 - 8(3)^2 + 5(3) - 7$$

$$f(3) = 2(81) - 8(9) + 15 - 7$$

$$f(3) = 162 - 72 + 15 - 7 = \boxed{98}$$