Pre-Calculus Name: Key!

Review for Final Exam

From Chapter 1

1) Find the domain & range:

a) $f(x) = -(x-3)^2 - 4$

vertex (3,-4)
parabola down

D: $(-\infty, \infty)$

R: $(-\infty, -4]$

b) $f(x) = 2\sqrt{9-(x-3)^2}$

x^2-6x+9

$9-(x-3)^2 \geq 0$
$-(x-3)^2 \geq -9$
$(x-3)^2 \leq 9$
$x^2-6x+9 \leq 9$
$x^2-6x \leq 0$
$x(x-6) \leq 0$

$+ \quad - \quad +$; 0, 6

D: $[0, 6]$

R: $[0, 6]$

c) $f(x) = |3\sqrt{x+8} - 4|$

$\sqrt{x+8} \geq 0$
$x \geq -8$
absolute value

D: $[-8, 0)$

R: $[0, \infty)$

From Chapter 1 & 2

2) Identify the increasing/decreasing intervals, local & absolute max/min, and state the end behavior:

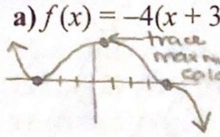
a) $f(x) = -4(x+3)^2(x-4)^3$

bounce at -3 flatten at 4

odd + negative

trace max with calculator

(-.2, 2323.4)



Inc: $(-3, -0.2)$

Dec: $(-\infty, -3) \cup (-0.2, \infty)$

local max: 2323.4 local min: 0

abs max: none abs min: none

LEB: $\lim_{x \to -\infty} f(x) = \infty$ REB: $\lim_{x \to \infty} f(x) = -\infty$

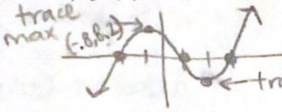
b) $f(x) = (x-3)(x-1)(x+2)$

cross through at 3, 1, -2

odd +

trace max (-.8, 8.2)

trace min (2.1, -4.1)



Inc: $(-\infty, -0.8) \cup (2.1, \infty)$

Dec: $(-0.8, 2.1)$

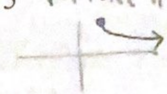
local max: 8.2 local min: -4.1

abs max: none abs min: none

LEB: $\lim_{x \to -\infty} f(x) = -\infty$ REB: $\lim_{x \to \infty} f(x) = \infty$

c) $f(x) = -2\sqrt{x-4} + 5$

vertex (4,5)



Inc: none

Dec: $[4, \infty)$

local max: 5 local min: none

abs max: 5 abs min: none

LEB: $\lim_{x \to -\infty} f(x) =$ DNE REB: $\lim_{x \to \infty} f(x) = -\infty$

From Chapter 2

3) Identify the asymptotes and write limit statements for each of the following:

a) $f(x) = \dfrac{2(x+2)}{x-5}$

$\frac{+}{=}$ $\frac{-}{+}$ $\frac{+}{+}$ same degree L.C. ratio

-3 -2 0 2 5 6

V.A. $x=5$

Zeros: $(-2, 0)$

HA: $y=2$ VA: $x=5$

end behav

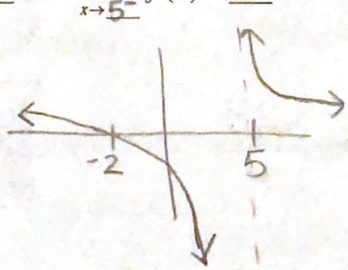
$\lim_{x \to \infty} f(x) = 2^+$

$\lim_{x \to -\infty} f(x) = 2^-$

$\lim_{x \to 5^+} f(x) = \infty$

$\lim_{x \to 5^-} f(x) = -\infty$

V.A.



b) $y = \dfrac{-2}{x+3}$ bigger on bottom

$\frac{-}{=}$ $\frac{-}{+}$

-5 -3 0

HA: $y=0$ VA: $x=-3$

end behavior

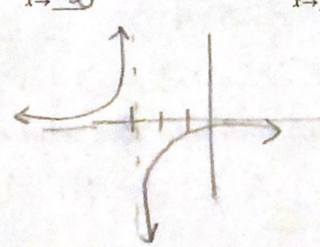
$\lim_{x \to \infty} f(x) = 0^-$

$\lim_{x \to -\infty} f(x) = 0^+$

$\lim_{x \to -3^-} f(x) = \infty$

$\lim_{x \to -3^+} f(x) = -\infty$

V.A.



From Chapter 3

4) Write an exponential model and find the requested information for each of the following:

a) The number of students taking AP Calculus at HSHS increases at a rate of 23% each year. If 22 students took the class at time $t = 0$, how many will take it after 5 years? How long will it take to reach enrollment of 100 students in AP Calculus?

Model: $y = 22(1.23)^x$

1st answer: 61 students $y = 22(1.23)^5$

2nd answer: 7.3 years (the 8th yr) $100 = 22(1.23)^x \to \log_{1.23}\frac{50}{11} = x$; $\frac{50}{11} = 1.23^x$

b) The value of a recently purchased car decreases at a rate of 6% each year. I the purchase price of the car was \$20,890, how long will it take to reach half of its original purchase price? How much will the car be worth after 3 years?

Model: $y = 20890(.94)^x$

1st answer: in the 11th yr $10445 = 20890(.94)^x$; $.5 = .94^x$; $\log_{.94}.5 = x$; $x = 11.2$

2nd answer: \$17350.90 $y = 20890(.94)^3$

c) A certain element has a half life of 29 days. If 37 grams of this element were present initially, how much will remain after 198 days? How long will it take for half the original sample to remain?

Model: $y = 37\left(\frac{1}{2}\right)^{\frac{x}{29}}$

1st answer: 0.3258 grams 2nd answer: 29 days

$y = 37\left(\frac{1}{2}\right)^{\frac{198}{29}}$

$18.5 = 37\left(\frac{1}{2}\right)^{\frac{x}{29}} \to .5 = .5^{\frac{x}{29}}$; $\log_{.5}.5 = \frac{x}{29}$; $1 = \frac{x}{29}$

From Chapter 3

5) Given the logistic growth model below identify the requested information.

a) $y = \dfrac{207}{1+8e^{-t}}$ ← c

$f(x) = \dfrac{c \leftarrow \text{max pop}}{1 + a \cdot b^x}$ (a ← initial, b ← ratio)

$\frac{207}{1+8e^0} = 23$

Equations of the H.A.: $y = 0$ & $y = 207$

Initial value: 23

Maximum sustainable population: 207

b) $f(x) = \dfrac{4000}{1+399e^{-2t}}$

$\frac{4000}{1+399e^{-2(0)}} = 10$

Equations of the H.A.: $y = 0$ & $y = 4000$

Initial value: 10

Maximum sustainable population: 4000

From Chapter 3

6) Solve each of the following equations for the EXACT solution (use calculator to verify only):

a) $9^x = 4^{5x}$

$x\log 9 = (5x)\log 4$

$x\log 9 - (5x)\log 4 = 0$

$\frac{x(\log 9 - 5\log 4)}{\log 9 - 5\log 4} = \frac{0}{\log 9 - 5\log 4}$

$x =$ 0

b) $17^x \cdot \dfrac{4}{17^2} = 2^{6x}$

$\frac{17^x \cdot 4 \cdot 17^{-2}}{4} = \frac{2^{6x}}{4}$

$17^x \cdot 17^{-2} = \frac{2^{6x}}{2^2}$

$17^{x-2} = 2^{6x-2}$

$(x-2)\log 17 = (6x-2)\log 2$

$x\log 17 - 2\log 17 = (6x)\log 2 - 2\log 2$

$x\log 17 - (6x)\log 2 = 2\log 17 - 2\log 2$

$x(\log 17 - 6\log 2) = 2\log 17 - 2\log 2$

$x\left(\log\left(\frac{17}{2^6}\right)\right) = \log\left(\frac{17^2}{2^2}\right)$

$x = \dfrac{\log\left(\frac{17^2}{4}\right)}{\log\left(\frac{17}{64}\right)}$

$x =$ −3.23

From Chapter 4 (calculator)

7) Evaluate each of the following:

a) $\tan\left(\frac{4\pi}{5}\right)$ -0.73

b) $\cot\left(\frac{4\pi}{5}\right)$ $\frac{1}{\tan}$ -1.38

c) $\sin\left(\frac{13\pi}{7}\right)$ -0.44

d) $\csc\left(\frac{13\pi}{7}\right)$ $\frac{1}{\sin}$ -2.30

e) $\sec 67°$ 2.56

$\frac{2\pi}{8}$ $\frac{2\pi}{4}$ $\frac{\pi}{4}$ $\frac{\pi}{2}$

From Chapter 4 & Chapter 5

8) Solve each of the following trigonometric equations on the interval [0, 2π). (Round to nearest hundredth)

a) $\sin\theta = \frac{2}{3}$

$\sin^{-1}\left(\frac{2}{3}\right)=\theta$

$\pi - .73$

$\theta = \underline{0.73}$

2.411

b) $\sec\theta = -7$ $2\pi - 1.71$

$\frac{1}{\cos\theta} = -7$

$1 = -7\cos\theta$

$\cos\theta = -\frac{1}{7}$

$\cos^{-1}\left(-\frac{1}{7}\right)=\theta$

$\theta = \underline{1.71}$

4.57

c) $12\sin^2 x + 17\sin x = 7$ $\pi - 0.34$

$12\sin^2 x + 17\sin x - 7 = 0$

$12x^2 + 17x - 7$ $(4\sin x + 7)(3\sin x - 1) = 0$

$x^2 + 17x - 84$ $\sin x = -\frac{7}{4}$ $\sin x = \frac{1}{3}$

$\frac{(x+21)(x-4)}{12} = (4x+7)(3x-1)$

$\sin^{-1}\left(-\frac{7}{4}\right) = x$ DNE $\sin^{-1}\left(\frac{1}{3}\right) = x$

$x = \underline{0.34}$

2.80

d) $2 + 2\sin x = 3\cos^2 x$

$2 + 2\sin x - 3\cos^2 x = 0$

$2 + 2\sin x - 3(1 - \sin^2 x) = 0$

$2 + 2\sin x - 3 + 3\sin^2 x = 0$

$3\sin^2 x + 2\sin x - 1 = 0$ $3x^2 + 2x - 1$

$(\sin x + 1)(3\sin x - 1) = 0$ $x^2 + 2x - 3$

$\frac{(x+3)(x-1)}{3}$

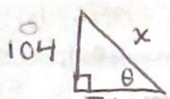
$x = \underline{3\pi/2,\ 0.34,\ 2.80}$

$\sin x = -1$ $\sin x = \frac{1}{3}$

$\sin^{-1}\left(\frac{1}{3}\right) = x$

From Chapter 4

9) A ladder leans against a wall. The base of the ladder is 71in from the bottom of the wall, and reaches a height of 104in. Determine the length of the ladder and the angle of elevation created by the ladder and the floor.



$\tan^{-1}\left(\frac{104}{71}\right) = \theta$

$\theta = 55.7°$

$104^2 + 71^2 = x^2$

Length of ladder = 125.9 inches

$\cos 55.7 = \frac{71}{x}$

∠of elevation = 55.7°

From Chapter 4

10) A bike has wheels with a radius of 16in. If the wheels are rotating at 47rpm determine the speed of the bike in mph (5280 ft = 1 mi)

63360 in = 1 mi

$r = 16$ $\omega = \frac{47\text{ rev}}{1\text{ min}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} = 94\pi\text{ rad/min}$

$V = \omega r$ $V = 94\pi(16) = 1504\pi\text{ in/min}$

$\frac{1504\pi\text{ in}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{1\text{ mile}}{63360\text{ in}} = \frac{90240\pi}{63360}$

speed = 1.42π mph or 4.47 mph

From Chapter 4

11) Given the information regarding arc length (s), radius (r), and the central angle (θ) fill in the table:

$S = r\theta$ ← radians, ↑ arclength, ↑ radius

s	R	θ
4π	14 units	2π/7
25π m	55m	5π/11
39π cm	71cm	39π/71

$4\pi = r\left(\frac{2\pi}{7}\right)$

$\frac{28\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$14 = r$

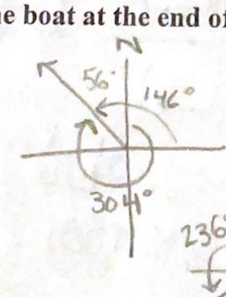
$S = 55\left(\frac{5\pi}{11}\right)$

$S = \frac{275\pi}{11}$

$\frac{39\pi}{71} = \frac{71\theta}{71}$

From Chapter 4

12) A boat travels on a bearing of 304° at 32knots for 3.4 hours. If the boat then changes direction to 214° and slows to 30knots for 2.1 hours what is the distance of the boat from the start point to the end point? What is the bearing of the boat at the end of its trip?

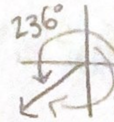


$V = \frac{32\text{ knot}}{3.4\text{ hr}} \approx 9.4\text{ knots/hr}$

$\langle 9.4\cos 146°, 9.4\sin 146° \rangle$

Distance from start to finish: 125.72 n.mi.

Bearing of boat at end of trip: 273.93 °



$V = \frac{30\text{ knot}}{2.1\text{ hr}} \approx 14.3\text{ knots/hr}$

$\langle 14.3\cos 236°, 14.3\sin 236° \rangle$

******Know the Law of Sines and Law of Cosines formulas and how to use them!!!****

$\langle -15.79, -6.58 \rangle$ $\theta = 22.62$ $+180$ 202.6

From Chapter 6

13) A plane travels on a bearing of 219° at 400mph. If a wind is blowing at a bearing of 211° at 45mph write a vector representing the velocity produced by the plane alone, a vector representing the velocity of the wind alone, and the resultant velocity representing the actual velocity of the plane. Then determine the actual speed of the plane and the direction angle of the plane (not the bearing).

231° 239°

p = ⟨-251.73, -310.86⟩ P ⟨400cos 231°, 400sin 231°⟩

w = ⟨-23.18, -38.57⟩ W ⟨45cos 239°, 45sin 239°⟩

v = ⟨-274.90, -349.43⟩ actual speed = 444.6 mph θ = 231.8°

$\sqrt{(-274.90)^2+(-349.43)^2}$

$\tan^{-1}\left(\frac{-349.43}{-274.9}\right)=\theta$

θ = 51.8 + 180

From Chapter 6

14) Given vector v = ⟨−3, −11⟩ & u = ⟨−2, 7⟩ find $\text{proj}_v u$ and then write u as the sum of two orthogonal vectors (one of which is $\text{proj}_v u$).

$\text{proj}_v u = \left\langle \frac{-213}{130}, \frac{-781}{130} \right\rangle$

$\text{proj}_v u = \left(\frac{u \cdot v}{|v|^2}\right) v \Rightarrow \frac{-71}{130}\langle -3,-11\rangle = \left\langle \frac{-213}{130}, \frac{-781}{130}\right\rangle$

$u = \left\langle \frac{-213}{130}, \frac{-781}{130} \right\rangle + \left\langle \frac{-47}{130}, \frac{1691}{130} \right\rangle$

$u \cdot v = (-3 \cdot -2) + (-11 \cdot 7) = 6 + -77 = -71$

$|v| = \sqrt{(-3)^2+(-11)^2} = \sqrt{130}$

$U = W_1 + W_2$

$|v|^2 = 130$

$U - W_1 = \langle -2, 7\rangle - \left\langle \frac{-213}{130}, \frac{-781}{130}\right\rangle$

From Chapter 9

15) In an arithmetic sequence $a_3 = 54099$ and $a_7 = 53655$. Write an explicit and recursive definition of the sequence, find a_{18} and the sum of the first 18 terms.

$a_n = -111n + 54432$ (explicit)

$a_{18} = 52434$

$a_n = a_1 + d(n-1)$

54099 53655
a_3 a_4 a_5 a_6 a_7
4 steps

$a_7 = a_3 + d(4)$

$a_1 = 54321$ $a_n = a_{n-1} - 111$ (recursive)

$S_{18} = 960795$

$53655 = 54099 + 4d$

$-444 = 4d$

$d = -111$

$\frac{18}{2}(54321+52434)$

$a_n = 54321 - 111(n-1)$

$54321 - 111n + 111$

$-111n + 54432$

$a_3 = a_1 - 111(n-1)$

$54099 = a_1 - 111(2)$

$54321 = a_1$

From Chapter 9

16) In a geometric sequence $g_4 = 16807$ and $g_9 = 1$. Write an explicit & recursive definition. Find g_{11} and the sum of the first 11 terms. If the series converges find the sum of the infinite sequence.

$g_n = 5764801\left(\frac{1}{7}\right)^{n-1}$ (explicit)

$g_{11} = \frac{1}{49}$

$\frac{a_1}{1-r}$ $\frac{5764801}{1-\frac{1}{7}}$

$g_1 = 5764801$ $g_n = a_{n-1} \cdot \frac{1}{7}$ (recursive)

$S_{11} = 6725601.163$

$S = 5764801\left(\frac{1-\frac{1}{7}^{11}}{1-\frac{1}{7}}\right)$

$S = 6725601.167$

$g_9 = g_4(r)^{n-1}$ (5)

$1 = 16807(r)^5$

$\frac{1}{16807} = r^5$

$r = \frac{1}{7}$

$g_4 = g_1\left(\frac{1}{7}\right)^3$

$16807 = g_1\left(\frac{1}{7}\right)^3$

$g_1 = 5764801$

From Chapter 1

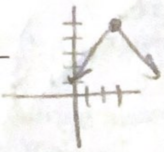
17) Identify the transformations applied to each of the 12 basic functions below, then state the domain, range, and whether it is or is not one-to-one:

a) $f(x) = -3|x-3| + 5$

transformations:

vertical flip, vertical stretch sf 3,

right 3, up 5



D: $(-\infty, \infty)$

R: $(-\infty, 5]$

1-to-1? no

b) $f(x) = \dfrac{7}{2-x}$ $\quad \dfrac{7}{-x+2} = \dfrac{7}{-(x-2)}$

$x \neq 2$

transformations:

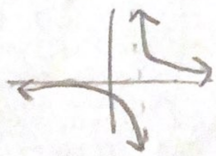
vertical stretch sf 7

right 2, horizontal flip

D: $(-\infty, 2) \cup (2, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

1-to-1? yes



c) $f(x) = -\sqrt{4x} + 11$ *

transformations:

up 11, vertical flip,

horizontal stretch sf $\frac{1}{4}$

D: $[0, \infty)$

R: $(-\infty, 11]$

1-to-1? yes

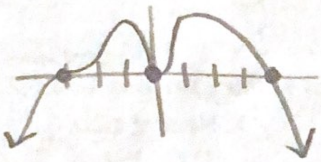
From Chapter 2

18) Sketch each of the following polynomials (include all intercepts) and write a statement for their end behavior: bounce flatten cross

(Hint – Rational Root Theorem)

a) $f(x) = -x^2(x+3)^3(x-4)$

even negative



x-int(s): 0, -3, 4

x=0

y-int: (0,0)

$\lim_{x\to-\infty} f(x) = -\infty$ $\quad \lim_{x\to\infty} f(x) = -\infty$

b) $f(x) = x^3 + x^2 - 16x - 16$

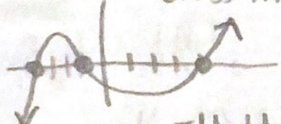
$x^2(x+1) - 16(x+1)$

odd + positive

$(x^2-16)(x+1)$

$(x+4)(x-4)(x+1)$

cross through



x-int(s): -4, 4, -1

y-int: (0, -16)

$\lim_{x\to\infty} f(x) = -\infty$ $\quad \lim_{x\to-\infty} f(x) = \infty$

c) $f(x) = x^3 - 7x^2 + 11x - 5$ $\quad \pm1, \pm5$

q ±1 p

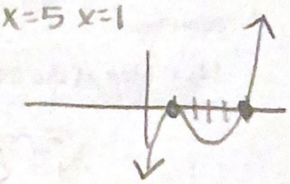
$\frac{p}{q} = \pm1, \pm5$

$x^2 - 6x + 5$ odd +

$(x-5)(x-1)$

$x=5 \quad x=1$

1 | 1 -7 11 -5
↓ 1 -6 5
1 -6 5 0



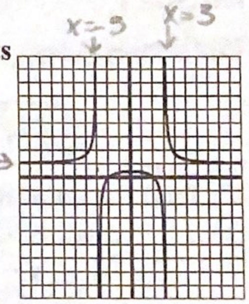
x-int(s): 1, 5

y-int: (0, -5)

$\lim_{x\to-\infty} f(x) = -\infty$ $\quad \lim_{x\to\infty} f(x) = \infty$

From Chapter 2

19) Use the graph of $f(x)$ at the right to complete each of the following limit statements



$\lim_{x\to-\infty} f(x) =$ 1

$\lim_{x\to-3^-} f(x) =$ ∞

$\lim_{x\to-3^+} f(x) =$ $-\infty$

$\lim_{x\to3^-} f(x) =$ $-\infty$

$\lim_{x\to3^+} f(x) =$ ∞

$\lim_{x\to\infty} f(x) =$ 1

x^2-x-12 $(x-\frac{4}{3})(x+\frac{3}{3})$ $(4x-10)(x+1) \to (3x+4)(3x-4)(x+1)$ $9x^2(x+1)-16(x+1)$ x^2-x-20 $(x-\frac{5}{4})(x+\frac{4}{4})$ $(4x-5)(x+1)$

From Chapter 2

20) Determine the holes, intercepts, & asymptotes for each of the following:

a) $f(x)=\dfrac{x^2-4}{x^2-9}$ $\dfrac{(x+2)(x-2)}{(x+3)(x-3)}$

Hole(s): (,) (,)

x-int: (-2, 0) (2, 0)

y-int: (0, $4/9$)

Eqs of ALL Asymptotes: $x=3$ $x=-3$ $y=1$

b) $f(x)=\dfrac{3x^2-x-4}{9x^3+9x^2-16x-16}$

Hole(s): (-1, 1) ($\frac{4}{3}$, $\frac{1}{8}$)

x-int: ($-\frac{4}{3}$, 0) (,)

y-int: (0, $\frac{1}{4}$)

Eqs of ALL Asymptotes: $y=0$ $x=-\frac{4}{3}$

$\dfrac{\cancel{(3x-4)}\cancel{(x+1)}}{(3x+4)\cancel{(3x-4)}\cancel{(x+1)}}=\dfrac{1}{3x+4}$

c) $f(x)=\dfrac{4x^2-x-5}{x-3}$

Hole(s): (,) (,)

x-int: ($\frac{5}{4}$, 0) (-1, 0)

y-int: (0, $5/3$)

Eqs of ALL Asymptotes: $x=3$ $y=4x+11$

$3 \,|\, 4 \;\; -1 \;\; -5$
$\;\;\; 12 \;\; 33$
$4 \;\; 11 \;\; 28$
$4x+11$

From Chapter 3

21) Which of the following are equivalent? ✓ *i.* $\frac{1}{2}+\log 3$ ✓ *ii.* $\frac{1}{2}\log 90$ ✓ *iii.* $\log 3\sqrt{10}$

all

$\log 90^{\frac{1}{2}}$
$\log\sqrt{90} \frac{9}{10}=\log 3\sqrt{10}$
$\log 3+\log\sqrt{10}$
$10^x=\sqrt{10}$
$10^x=10^{\frac{1}{2}}$

From Chapter 3

22) Simplify: $\dfrac{\log 27}{\log 81}$ **A.** $\log\frac{1}{3}$ **B.** $\frac{1}{3}$ **C.** $\log 27-\log 81$ **D.** $\frac{3}{4}$ **E.** Cannot determine without calculator

$\log_{81}27$
$81^x=27$ $3^{4x}=3^3$
$4x=3$
$x=3/4$

From Chapter 3

23) Which of the following is the value of **$-\log 0.00001$?** **A.** -5 **B.** -4 **C.** $\frac{1}{4}$ **D.** $\frac{1}{5}$ **E.** None of these

$=5$

$\frac{1}{10}=.1$ $\frac{1}{100}=.01$ $\frac{1}{1000}=.001$ $\frac{1}{10{,}000}=.0001$ $\frac{1}{100{,}000}=.00001$
10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5}

From Chapter 3

24) Which of the following is the value of $\log_4\dfrac{4}{\sqrt[6]{64}}$? **A.** $-\frac{1}{2}$ **B.** $\frac{1}{2}$ **C.** $\frac{1}{3}$ **D.** $-\frac{1}{3}$ **E.** None of these

$\dfrac{4}{\sqrt[6]{4^3}}=\dfrac{4^1}{4^{3/6}}=\dfrac{4^1}{4^{1/2}}=4^{\frac{1}{2}}=2$ $\log_4 2=x$ $4^x=2$

From Chapter 3

25) Which of the following is the value of $-\log_{\frac{1}{3}}243$? **A.** $-1/5$ **B.** -5 **C.** 5 **D.** $1/5$ **E.** None of these

$3^x=243$

From Chapter 3

26) Given that $\log_{\sqrt[5]{64}}x=\dfrac{5}{3}$**, what is the value of x?** **A.** 81 **B.** 3/2 **C.** 9 **D.** 36 **E.** None of these

$\sqrt[5]{64}^{\frac{5}{3}}=x$ $(64^{\frac{1}{5}})^{\frac{5}{3}}=64^{\frac{5}{15}}=64^{\frac{1}{3}}=\sqrt[3]{64}=4$

From Chapter 4

27) Find the amplitude, period, phase shift, and vertical shift of each of the following:

a) $f(x)=5\sin(3x-\pi)+4$

$\underset{a}{5}\sin(\underset{b}{3}(x-\frac{\pi}{3}))+4$

amp = 5 pd = $\frac{2\pi}{3}$

P.S. = right $\frac{\pi}{3}$ V.S. = ↑4

period $=\frac{2\pi}{B}$

b) $f(x)=-3\cos(½x-\pi/2)-1$

vertical flip $\underset{A}{-3}\cos(\underset{B}{\frac{1}{2}}(x-\pi))-1$

amp = 3 pd = 4π

P.S. = right π V.S. = ↓1

period $=\frac{2\pi}{B}$ $\frac{2\pi}{\frac{1}{2}}$

$x^2+y^2=r^2$ $x=r\cos\theta$ $y=r\sin\theta$

From Chapter 6

28) Write the rectangular equation as a polar equation $2x^2+2y^2=5y$

$2(x^2+y^2)=5y$

$2r^2=5(r\sin\theta)$

$2r^2-5r\sin\theta=0$

$\cancel{r}(2r-5\sin\theta)=0$

$2r=5\sin\theta$

$\boxed{r=\frac{5\sin\theta}{2}}$

From Chapter 6

29) Eliminate the parameter and describe the resulting graph:

a) $x=4\cos^2\theta$ & $y=2\sin\theta$

$\frac{x}{4}=\cos^2\theta$ $\left(\frac{y}{2}\right)^2=(\sin\theta)^2$ $x+y^2=4$

$\frac{x}{4}+\frac{y^2}{4}=1$ $y^2=4-x$

$\boxed{y=\pm\sqrt{4-x}}$

b) $x=e^t$ & $y=e^{-t}$

$\ln x=t$ $\ln y=-t$

$-\ln y=\ln x$

$0=\ln x+\ln y$

$0=\ln(xy)$

$10^0=xy$

$1=xy$

$y=\frac{1}{x}$

From Chapter 6

30) Convert the following polar points to rectangular coordinates.

a) $\left(6,\frac{\pi}{2}\right)$ r θ

$\boxed{(0,6)}$

b) $\left(-1,\frac{7\pi}{4}\right)$

$\boxed{\left(-\frac{\sqrt2}{2},\frac{\sqrt2}{2}\right)}$

From Chapter 6

31) Convert the following rectangular points to polar coordinates

a) $(-3, 3)$

$(-3)^2+(3)^2=r^2$

$9+9>18=r^2$

$r=3\sqrt2$

$\tan^{-1}\left(\frac{3}{-3}\right)=\theta$

$\theta=\frac{3\pi}{4}$

$\boxed{\left(3\sqrt2,\frac{3\pi}{4}\right)}$

b) $\left(\frac{1}{2},-\frac{\sqrt3}{2}\right)$

$\boxed{\left(1,\frac{5\pi}{3}\right)}$

From Chapter 4

32) Name one positive and one negative angle co-terminal with each of the following angles:

a) $\frac{2\pi}{3}$ $\pm\frac{6\pi}{3}$

(+) $8\pi/3$ radians (–) $-4\pi/3$ radians

b) $315°$ ± 360

(+) 675 ° (–) $-45°$ °

From Chapter 4

33) Evaluate each of the following:

a) $\cot\left(\frac{5\pi}{4}\right)=$ 1 $\frac{\cos}{\sin}$

b) $\sin(330°)=$ $\frac{-1}{2}$

c) $\sec\left(\frac{5\pi}{6}\right)=$ $\frac{-2}{\sqrt3}$ $\frac{1}{\cos}$

d) $\cos(-300°)=$ $\frac{1}{2}$

e) $\csc\left(-\frac{3\pi}{2}\right)=$ 1 $\frac{1}{\sin}$

f) $\sec(90°)=$ undefined $\frac{1}{\cos}$

g) $\tan(5\pi)=$ 0 $\tan(\pi)$

From Chapter 4

34) Find the exact value of each of the following, write all angle measures in radians, if the expression is undefined write "undefined" :

a) $\arctan(1)$ $\frac{\pi}{4}$

b) $\sin^{-1}\left(-\frac{1}{2}\right)$ $\frac{-\pi}{6}$

☆ **c)** $\cos^{-1}\left(\frac{2}{\sqrt3}\right)>1$ undefined

d) $\arcsin\left(\frac{\sqrt2}{2}\right)$ $\frac{\pi}{4}$

From Chapter 4

35) Determine the quadrant in which θ lies given that $\sin\theta>0$ & $\sec\theta<0$ $\frac{1}{\cos\theta}$ **Quadrant:** II

I + II II + III