Unit 2 – Polynomial Functions Review

Part of Polynomial Functions

1) Determine whether each of the following is a polynomial. If it is state the degree & leading coefficient (if not put "n/a"):

a. $f(x)=5x^{3.2}+4x-3$

This not a polynomial. (is/is not)
Degree = __ & Leading coeff. = __

b. $f(x)=7x^2-3x+8$

This is a polynomial. (is/is not)
Degree = 2 & Leading coeff. = 7

c. $f(x)=\dfrac{7x+5}{3x}$ $\quad \cancel{\frac{7x}{3x}}+\frac{5}{3x}$

This not a polynomial. (is/is not)
Degree = __ & Leading coeff. = __

d. $f(x)=\dfrac{7x+5}{3}$ $\quad \frac{7}{3}x+\frac{5}{3}$

This is a polynomial. (is/is not)
Degree = 1 & Leading coeff. = 7/3

e. $f(x)=\sqrt[3]{18x^3+4x^2+10}$

This not a polynomial. (is/is not)
Degree = __ & Leading coeff. = __

f. $f(x)=5x^4-4x^6+3x^3+11x-2$

This is a polynomial. (is/is not)
Degree = 6 & Leading coeff. = -4

2) For each of the following polynomials state the degree, leading coefficient, & the constant term:

a. $f(x) = 5x^6 - 4x^3 + 2x - 5$

Degree: 6
Leading coefficient: 5
Constant: -5

b. $f(x) = 5x^2 - 3x^4 + 7$

Degree: 4
Leading coefficient: -3
Constant: 7

$-3x^4+5x^2+7$

c. $f(x)=3(x-2)^2(x+5)(2x-1)$

Degree: 4
Leading coefficient: 3
Constant: -60

$3(0-2)^2(0+5)(0-1)=3\cdot4\cdot5\cdot-1=-60$

d. $f(x)=-4x(x+1)(x-3)^3$

Degree: 5
Leading coefficient: -4
Constant: 0

Graphs of Power Functions

3) For each of the following determine whether it is a radical, rational, or polynomial function:

a. $f(x)=\frac{1}{2}x^{\frac{3}{4}}$
radical/rational/polynomial
$\frac{1}{2}\sqrt[4]{x^3}$

b. $f(x)=-3x^{-5}$ $\quad \frac{-3}{x^5}$
radical/rational/polynomial

c. $f(x)=2x^{\frac{2}{5}}$
radical/rational/polynomial
$2\sqrt[5]{x^2}$

d. $f(x)=2x^{\frac{9}{3}}$
radical/rational/polynomial
$2x^3$

4) For each of the following determine which quadrants contain the graph of the given function:

a. $f(x)=\frac{1}{2}x^{\frac{3}{4}}$ $\quad \frac{1}{2}\sqrt[4]{x^3}$

CIRCLE ONE
Quadrant I & III | Quadrant IV only
Quadrant II & IV | Quadrant I & II
Quadrant I only | Quadrant III & IV

b. $f(x)=-3x^{-5}$ $\quad -\frac{3}{x^5}$

CIRCLE ONE
Quadrant I & III | Quadrant IV only
Quadrant II & IV | Quadrant I & II
Quadrant I only | Quadrant III & IV

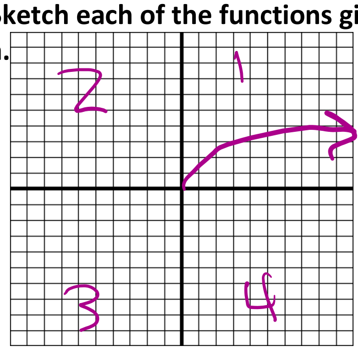
c. $f(x)=2x^{\frac{2}{5}}$ $\quad 2\sqrt[5]{x^2}$

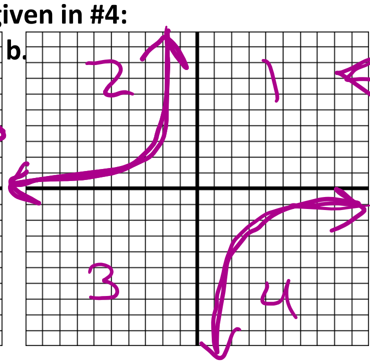
CIRCLE ONE
Quadrant I & III | Quadrant IV only
Quadrant II & IV | Quadrant I & II
Quadrant I only | Quadrant III & IV

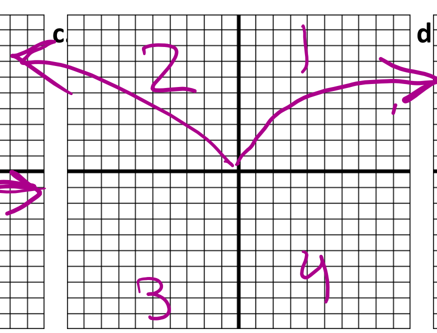
d. $f(x)=2x^{\frac{9}{3}}$ $\quad 2x^3$

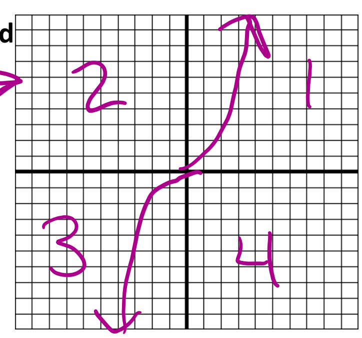
CIRCLE ONE
Quadrant I & III | Quadrant IV only
Quadrant II & IV | Quadrant I & II
Quadrant I only | Quadrant III & IV

Sketch each of the functions given in #4:

a.


b.


c


d


Review of Factoring

5) a) $u^6 - 12u^3 + 27$

$\boxed{(u^3-9)(u^3-3)}$

b) $2x^7 + 26x^4 + 72x$

gcf $2x(x^6+13x^3+36)$

$\boxed{2x(x^3+9)(x^3+4)}$

c) $3x^9 + 27x^5 + 60x$

$3x(x^8+9x^4+20)$

$\boxed{3x(x^4+4)(x^4+5)}$

d) $5x^2 + 20x + 15$

$5(x^2+4x+3)$

$\boxed{5(x+3)(x+1)}$

e) $3m^3 - 15m^2 - 150m$

$3m(m^2-5m-50)$

$\boxed{3m(m-10)(m+5)}$

f) $x^3 - 3x^2$

$\boxed{x^2(x-3)}$

g) $3x^3 - 15x^2 - 72x$

$3x(x^2-5x-24)$

$\boxed{3x(x-8)(x+3)}$

h) $6n^2 - 18n - 168$

$6(n^2-3n-28)$

$\boxed{6(n-7)(n+4)}$

i) $81a^3 + 24$

$3(27a^3+8)$

$\boxed{3(3a+2)(9a^2-6a+4)}$

Graphing Polynomial Functions

6) Identify the x-intercepts and the behavior at each. Identify the degree, leading coefficient as positive or negative, and end behavior or each function. Then, sketch each of the following:

a. $f(x) = (x-1)^3(x-2)^2$

Degree of the Polynomial: 5 odd

Leading Coefficient: +1

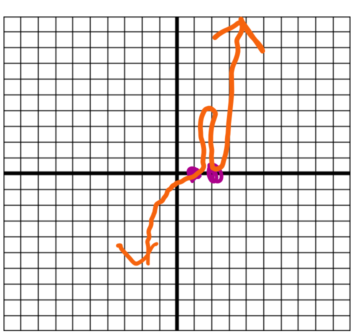
LEB: $\lim_{x\to-\infty} f(x) = -\infty$

REB: $\lim_{x\to\infty} f(x) = \infty$

x- intercepts & behavior:

x=1 flatten

x=2 bounce



b. $f(x) = -3(x+2)^2(x-4)^6$

Degree of the Polynomial: 8 even

Leading Coefficient: −3

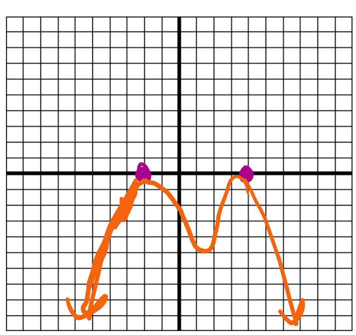
LEB: $\lim_{x\to-\infty} f(x) = -\infty$

REB: $\lim_{x\to\infty} f(x) = -\infty$

x- intercepts & behavior:

x=−2 bounce

x=4 bounce



c. $f(x) = -x^3(x+3)^2$

Degree of the Polynomial: 5 odd

Leading Coefficient: −1

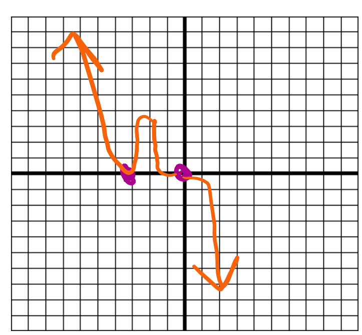
LEB: $\lim_{x\to-\infty} f(x) = \infty$

REB: $\lim_{x\to\infty} f(x) = -\infty$

x- intercepts & behavior:

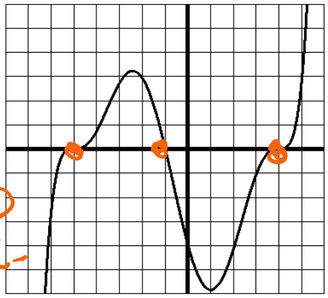
x=0 flatten

x=−3 bounce



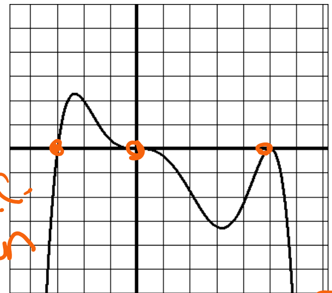
7) Given each graph below write a possible linear factorization for the polynomial:

a. *f(x)=* $(x+5)^3(x+1)(x-4)^3$ **b.** *f(x)=* $-x^3(x+3)(x-5)^2$ **c.** *f(x)=* $-(x+3)^3(x-1)^2$



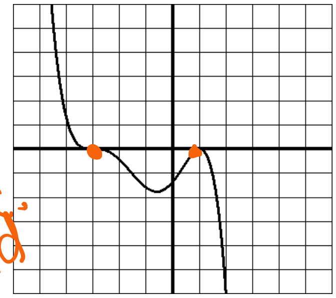
ODD +L.C.

$x=-5$ flatten, $x=-1$ cross, $x=4$ flatten



−L.C. even

$x=-3$ cross, $x=0$ flatten, $x=5$ bounce



−L.C. odd

$x=-3$ flatten, $x=1$ bounce

Finding Roots of Polynomial Functions

7) Find the ALL EXACT zeros by graphing and using synthetic division if needed.

a. $f(x)=x^4-5x^2+4$

$(x^2-4)(x^2-1)$

$(x+2)(x-2)(x+1)(x-1)$

b. $f(x)=x^4-2x^3+8x-16$

$x^3(x-2)+8(x-2)$

$(x^3+8)(x-2)$

$(x+2)(x^2-2x+4)(x-2)$

OR

2	1	−2	0	8	−16
	↓	2	0	0	16
−2	1	0	0	8	0
	↓	−2	4	−8	
	1	−2	4	0	

$a=1\ b=-2\ c=4$

$x=\dfrac{2\pm\sqrt{4-4(1)(4)}}{2}$

$x=\dfrac{2\pm\sqrt{-12}}{2}$

$\sqrt{12}$, $4\cdot3$, $2\sqrt{3}$

$x=\dfrac{2\pm2i\sqrt{3}}{2}$

c. $f(x)=81x^4-16$

$(9x^2)^2-4^2$

$(9x^2-4)(9x^2+4)$

$(3x+2)(3x-2)(9x^2+4)$

$9x^2+4=0$

$9x^2=-4$

$x^2=\frac{-4}{9}$

$x=\pm\frac{2}{3}i$

Factored Form of the Polynomial: $f(x)=(x+2)(x-2)(x+1)(x-1)$

Degree of the Polynomial: 4 even

Leading Coefficient: +1

Zero(s): $x=-2, x=2, x=-1, x=1$

LEB: $\lim_{x\to-\infty} f(x)=\infty$

REB: $\lim_{x\to\infty} f(x)=\infty$

Factored Form of the Polynomial: $f(x)=(x-2)(x+2)(x^2-2x+4)$

Degree of the Polynomial: 4 even

Leading Coefficient: +1

Zero(s): $x=2, x=-2, x=1+i\sqrt{3}$, $x=1-i\sqrt{3}$

LEB: $\lim_{x\to-\infty} f(x)=\infty$

REB: $\lim_{x\to\infty} f(x)=\infty$

Factored Form of the Polynomial: $f(x)=(3x+2)(3x-2)(9x^2+4)$

Degree of the Polynomial: 4 even

Leading Coefficient: +81

Zero(s): $x=-\frac{2}{3}, x=\frac{2}{3}, x=\frac{2}{3}i, x=-\frac{2}{3}i$

LEB: $\lim_{x\to-\infty} f(x)=\infty$

REB: $\lim_{x\to\infty} f(x)=\infty$

d. $f(x) = -4x^3 - 12x^2 + 36x + 108$

$-4((x^3+3x^2)-9x-27)$

$x^2(x+3)-9(x+3)$

$-4(x^2-9)(x+3)$

$-4(x+3)(x-3)(x+3)$

Factored Form of the Polynomial: $f(x) = -4(x+3)^2(x-3)$

Degree of the Polynomial: 3 (odd)

Leading Coefficient: -4

Zero(s): $x=-3$, $x=3$
bounce

LEB: $\lim_{x\to-\infty} f(x) = \infty$

REB: $\lim_{x\to\infty} f(x) = -\infty$

e. $f(x) = 3x^3 + x^2 - 21x - 7$

$x^2(3x+1)-7(3x+1)$

$(x^2-7)(3x+1)$

$x^2-7=0$

$x^2=7$

$x=\pm\sqrt{7}$

Factored Form of the Polynomial: $f(x) = (x^2-7)(3x+1)$

Degree of the Polynomial: 3 odd

Leading Coefficient: $+3$

Zero(s): $x=-\frac{1}{3}$, $x=\sqrt{7}$, $x=-\sqrt{7}$

LEB: $\lim_{x\to-\infty} f(x) = -\infty$

REB: $\lim_{x\to\infty} f(x) = \infty$

f. $f(x) = 4x^4 - 101x^2 + 25$

x^4-101x^2+100

$(x^2-\frac{100}{4})(x^2-\frac{1}{4})$

$(x^2-25)(4x^2-1)$

$(x+5)(x-5)(2x+1)(2x-1)$

Factored Form of the Polynomial: $f(x) = (x+5)(x-5)(2x+1)(2x-1)$

Degree of the Polynomial: 4 even

Leading Coefficient: $+4$

Zero(s): $x=-5$, $x=5$, $x=\frac{-1}{2}$, $x=\frac{1}{2}$

LEB: $\lim_{x\to-\infty} f(x) = \infty$

REB: $\lim_{x\to\infty} f(x) = \infty$

Complex Zeros of Polynomials

8) Find ALL of the complex zeros of the following polynomials: (REMEMBER WHAT THIS MEANS?)

a. $f(x) = x^3 - 6x^2 - 9x + 14$

$\boxed{x=-2 \quad x=1 \quad x=7}$

$(x+2)(x-1)(x-7)$

3 rational solutions

b. $f(x) = x^3 + 11x^2 + 5x + 55$

$x^2(x+11)+5(x+11)$

$(x^2+5)(x+11)$

$x^2+5=0$ $\boxed{x=-11}$

$x^2=-5$ 1 rational solution

$x=\pm\sqrt{-5}$

$x=\pm i\sqrt{5}$

$\boxed{x=i\sqrt{5}}$ $\boxed{x=-i\sqrt{5}}$

2 imaginary solutions

c. ~~$f(x) = x^4 + 3x^3 + 9x^2 + 12x + 20$~~

omit.

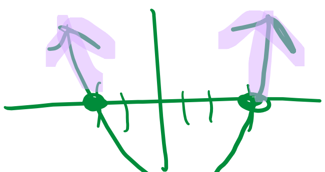
9) Solving Polynomial Inequalities-------------KEY POINTS TO REMEMBER

- **Factor 1st & Simplify if you can**
- **When using a SIGN CHART SHOW ALL WORK!!!**
- **If using a sketch of the polynomial to solve, then you MUST CLEARLY show the LABELED SKETCH**
- **Make SURE to state intervals which satisfy the inequality!**

even +

a) $(2x+4)(x-3) > 0$

x = -2 x = 3

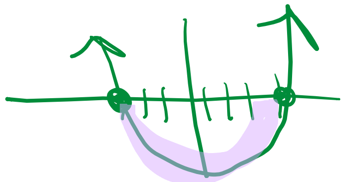


$\boxed{(-\infty,-2)\cup(3,\infty)}$

even +

b) $(x-4)(x+3) < 0$

x = 4 x = -3

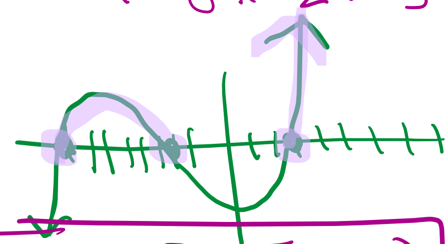


$\boxed{(-3,4)}$

odd +

c) $(x+8)(x+2)(x-3) \geq 0$

x = -8 x = -2 x = 3



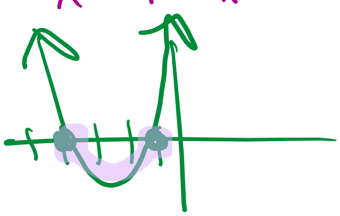
$\boxed{[-8,-2]\cup[3,\infty)}$

even +

d) $x^2+5x+4 \leq 0$

$(x+4)(x+1)$

x = -4 x = -1



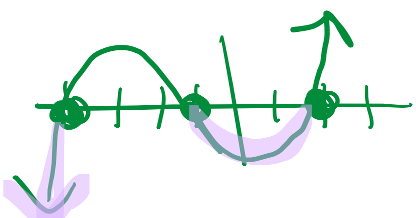
$[-4,-1]$

odd +

e) $x^3+3x^2-6x-8 < 0$

$(x+4)(x+1)(x-2) < 0$

x = -4 x = -1 x = 2

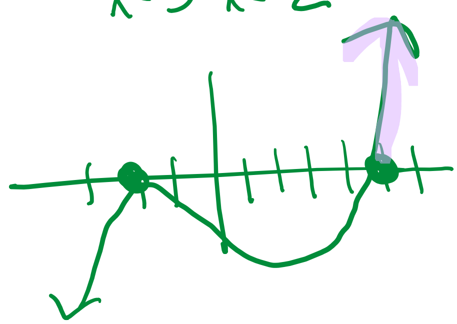


$(-\infty,-4)\cup(-1,2)$

odd +

f) $(x-5)^2(x+2) > 0$

x = 5 x = -2



$(5,\infty)$