Key

WHAT DID THE NINJA TURTLES SAY WHEN HANDED THE EXPRESSION $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$?

For each function evaluate $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

1) $f(x) = 3x + 2$

$$\lim_{h\to 0}\frac{3(x+h)+2-(3x+2)}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{3x}+3h+\cancel{2}-\cancel{3x}-\cancel{2}}{h}$$
$$=\boxed{3}$$

2) $f(x) = 4x - 3$

$$\lim_{x\to a}\frac{(4x-\cancel{3})-(4a-\cancel{3})}{x-a}$$
$$=\lim_{x\to a}\frac{4x-4a}{x-a}=\lim_{x\to a}\frac{4\cancel{(x-a)}}{\cancel{x-a}}$$
$$=\boxed{4}$$

3) $f(x) = x^2$

$$\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}=\boxed{2x}$$

4) $f(x) = x^2 - 5$

$$\lim_{h\to 0}\frac{[(x+h)^2-5]-(x^2-5)}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{5}-\cancel{x^2}+\cancel{5}}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}=\boxed{2x}$$

5) $f(x) = 3x^2 + x$

$$\lim_{h\to 0}\frac{[3(x+h)^2+(x+h)]-[3x^2+x]}{h}$$
$$\lim_{h\to 0}\frac{3(x^2+2xh+h^2)+\cancel{x}+h-3x^2-\cancel{x}}{h}$$
$$\lim_{h\to 0}\frac{\cancel{3x^2}+6xh+3h^2+h-\cancel{3x^2}}{h}$$
$$\lim_{h\to 0}\frac{h(6x+3h+1)}{h}=\boxed{6x+1}$$

6) $f(x) = x^3$

$$\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$$
$$\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}$$
$$\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$
$$=\boxed{3x^2}$$

7) $f(x) = 4x^2 + 2x - 7$

$$\lim_{h\to 0}\frac{[4(x+h)^2+2(x+h)-7]-[4x^2+2x-7]}{h}$$
$$\lim_{h\to 0}\frac{4(x^2+2xh+h^2)+\cancel{2x}+2h-\cancel{7}-4x^2-\cancel{2x}+\cancel{7}}{h}$$
$$\lim_{h\to 0}\frac{\cancel{4x^2}+8xh+4h^2+2h-\cancel{4x^2}}{h}$$
$$\lim_{h\to 0}\frac{\cancel{h}(8x+4h+2)}{\cancel{h}}=\boxed{8x+2}$$

8) $f(x) = x^4 + 1$

$$\lim_{h\to 0}\frac{((x+h)^4+1)-(x^4+1)}{h}$$
$$\lim_{h\to 0}\frac{\cancel{x^4}+4x^3h+6x^2h^2+4xh^3+h^4+\cancel{1}-\cancel{x^4}-\cancel{1}}{h}$$
$$\lim_{h\to 0}\frac{\cancel{h}(4x^3+6x^2h+4xh^2+h^3)}{\cancel{h}}$$
$$\boxed{4x^3}$$

Limits.

A. $f'(x) = 6x + 1$	**C.** $f'(x) = 4x + 2$	**D.** $f'(x) = 4x^3$	**E.** $f'(x) = 2x$	**F.** $f'(x) = 3x$
I. $f'(x) = 3$	**K.** $f'(x) = 4x$	**R.** $f'(x) = 3x^2$	**T.** $f'(x) = 8x + 2$	**V.** $f'(x) = 4$

D	E	R	I	V	A	T	I	V	E
8	3	6	1	2	5	7	1	2	4

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Name Key
Date

Derivatives Day 2

1. Which of the line(s) are tangent to the curve?

B

2. Which derivative is approximated by $\frac{\tan\left(\frac{\pi}{4}+0.0001\right)-1}{0.0001}$?

deriv of $\tan x$ @ $x = \pi/4$

Use the following figure

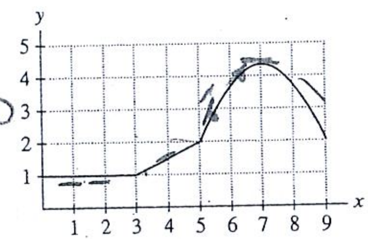
3. Determine $f'(a)$ for $a = 1, 2, 4, 7$.

$f'(1) = 0 \quad f'(2) = 0 \quad f'(4) = \frac{1}{2} \quad f'(7) = 0$

4. For which values of x is $f'(x) < 0$?

$(7, 9)$

5. Which is larger, $f'(5.5)$ or $f'(6.5)$? $f'(5.5)$



6. Show that $f'(3)$ does not exist.

$f'(3)$ from Left $= 0$ · $f'(3)$ from the right $\frac{1}{2}$ since the slopes are different $f'(3)$ DNE

7. The table shows that traffic speed, S, along a certain road (in km/h) varies as a function of traffic density, q (number of cars per km of road). Estimate $S'(80)$.

cars/Km
Km/hr

q (density)	60	70	80	90	100
S (speed)	72.5	67.5	63.5	60	56

$$S'(80) = \frac{60-67.5}{90-70} = \frac{-7.5}{20} = \boxed{-.375 \ \frac{\text{Km/hr}}{\text{cars/Km}}}$$

8. Find an equation of the tangent line at $x = 3$, assuming that $f(3) = 5$ and $f'(3) = 2$.

point slope

$$\boxed{y - 5 = 2(x-3)}$$

For 9-12, find $f'(a)$ for the given value of a.

9. $f(x) = x^2 + 9x, \quad a = 0$

10. $f(x) = x^2 + 9x, \quad a = 2$

11. $f(x) = 3x^2 + 4x + 2, \quad a = -1$

12. $f(x) = x^3, \quad a = 2$

Derivatives Day 2

9) $f'(0) = \lim_{x \to 0} \frac{x^2+9x-0}{x-0} = \lim_{x\to 0} \frac{\cancel{x}(x+9)}{\cancel{x}} = \boxed{9}$

10) $f'(2) = \lim_{x \to 2} \frac{x^2+9x-(4+18)}{x-2} = \lim_{x\to 2} \frac{x^2+9x-22}{x-2}$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+11)}{\cancel{x-2}} = \boxed{13}$

$3-4+2$

11) $f'(-1) = \lim_{h\to 0} \frac{3(-1+h)^2+4(-1+h)+2-(3(-1)^2+4(-1)+2)}{h}$

$= \lim_{h\to 0} \frac{3(1-2h+h^2)-4+4h+2-1}{h}$

$= \lim_{h\to 0} \frac{\cancel{3}-6h+3h^2\cancel{-4}+4h\cancel{+2}\cancel{-1}}{h}$

$= \lim_{h\to 0} \frac{-2h+3h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(-2+3h)}{\cancel{h}} = \boxed{-2}$

12) $f'(2) = \lim_{h\to 0} \frac{(2+h)^3-(2)^3}{h} = \lim_{h\to 0} \frac{\cancel{(2)^3}+3(2)^2h+3(2)h^2+h^3\cancel{-(2)^3}}{h}$

$= \lim_{h\to 0} \frac{12h+6h^2+h^3}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(12+6h+h^2)}{\cancel{h}} = 12$